

Closed-form analysis of multibranch switched diversity with noncoherent and differentially coherent detection

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SUMMARY

Exact *closed-form* analytical expressions are derived for the average bit error probability of multibranch switched diversity systems over independent and identically Nakagami- m distributed fading channels. Practical schemes that use noncoherent or differentially coherent symbol detection are considered. The general bit error probability expression derived in this paper includes as particular cases the following signaling formats: orthogonal binary signaling, correlated binary signaling, differential phase-shift keying, and differential quadrature phase-shift keying. Finally, we apply our analytical results to study the impact of the switching threshold selection on the system performance. Copyright © 2011 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Switched diversity has been thoroughly studied by communication theorists and engineers as an attempt for exploiting space diversity by simple practical systems [1–3]. Basically, this technique tracks a given diversity branch as long as channel quality stays above certain predefined threshold and switches to another branch when this condition fails. From the signal processing point of view, switched diversity is simpler to implement than maximal ratio combining, equal gain combining, or selection combining. The simplest and best studied switched diversity systems are switch-and-stay combining (SSC) and switch-and-examine combining (SEC) over independent and identically distributed (i.i.d.) channels. A comprehensive description of such systems can be found in [3, Chap. 9] and the references therein.

Besides, binary frequency-shift keying (FSK) signaling with noncoherent symbol detection is often adopted in practical SSC and SEC systems as a simple (low-complexity) modulation scheme. In such cases, signals can be chosen to be nonorthogonal at the transmitter to reduce bandwidth utilization, at the expense of certain performance degradation [4, Chap. 5]. Moreover, the performance of these systems can be improved with slightly higher complexity modulation schemes such as phase-shift keying (PSK) with differentially coherent detection. As simple modulation schemes, noncoherent and differentially coherent detection are of particular interest in low-complexity SEC receivers, which could be part of a multihop or relay network.

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This paper focuses on the performance analysis of SSC and SEC over i.i.d. Nakagami- m fading channels. In [5] and [6], an analytical framework was presented for the performance of coherent, noncoherent, and differentially coherent detection. Adopting the moment generating function (MGF) approach, results in [5] and [6] for the average bit error probability (BEP) were in the form of single finite integrals. In [7], a new analysis was performed to calculate these integrals in exact closed form for *coherent detection* (M-PSK, M-PAM, and M-QAM) in Nakagami- m fading channels with integer parameter m . However, to the best of the authors' knowledge, exact closed-form expressions for *noncoherent* and *differentially coherent* detection are not available in the literature. Only very recently, new results have appeared in [8] for the particular case of a dual-branch SSC system with noncoherent detection of nonorthogonal binary FSK under Rayleigh fading.

In this paper, a unifying closed-form BEP analysis is presented for *noncoherent* and *differentially coherent* detection in multibranch switched diversity systems over i.i.d. Nakagami- m fading channels. The analysis in this paper extends previous results obtained in [8], providing a threefold generalization: first, the number of branches is extended to an L -branch diversity system; second, the analysis is extended to the more general Nakagami- m fading model; and third, the BEP analysis includes other modulation schemes such as the differential quadrature PSK (DQPSK).[‡] Interestingly, recent mathematical results obtained in [9] for a class of incomplete cylindrical integrals, which have been used to analyze maximal ratio combining systems with *coherent* detection, are found to be crucial in the unified analysis that is presented in this paper. The general BEP expression derived here is in the form of a finite combination of Marcum Q , Bessel, and elementary functions, thus avoiding the need for numerical integration.

The remainder of this paper is structured as follows. Section 2 is devoted to characterize the statistics of the analyzed switched diversity systems. Then, exact and approximated closed-form expressions for the average BEP are derived in Section 3. Some numerical results are provided in Section 4, and finally, conclusions are drawn in Section 5.

2. CLASSICAL MULTIBRANCH SWITCH-AND-EXAMINE COMBINING STATISTICS

Let us assume a switched diversity system with L i.i.d. Nakagami- m branches. It is known that the output statistics of SSC does not depend on the number of diversity branches and that SEC has the same output statistics as SSC when $L = 2$ [6]. As noted in [7], the average BEP in SSC is obtained by setting $L = 2$ in SEC. Therefore, we restrict the analysis to the general L -branch SEC.

For SEC, the probability density function (PDF) of the instantaneous signal-to-noise ratio (SNR) per symbol γ_S at the output of the combiner is [3, eq. (9.341)]

$$f_{\gamma_S}(x) = \begin{cases} [F_\gamma(\gamma_T)]^{L-1} f_\gamma(x), & 0 \leq x < \gamma_T \\ \sum_{\ell=0}^{L-1} [F_\gamma(\gamma_T)]^\ell f_\gamma(x), & x \geq \gamma_T \end{cases}, \quad (1)$$

where f_γ and F_γ are the PDF and the cumulative distribution functions of the instantaneous SNR per symbol γ on each diversity branch, whereas γ_T denotes the switching threshold. The Nakagami- m distribution with integer m is considered for γ , which covers many cases of interest in practice, in particular, Rayleigh fading when $m = 1$. For an integer Nakagami m -parameter, it is well known that $F_\gamma(\gamma_T)$ and $f_\gamma(x)$ in Equation (1) are given by [3, table (9.5)] [10, eq. (8.352-2)]

$$F_\gamma(\gamma_T) = 1 - e^{-\frac{m}{\bar{\gamma}}\gamma_T} \sum_{\ell=0}^{m-1} \frac{\gamma_T^\ell}{(\ell)!} \left(\frac{m}{\bar{\gamma}}\right)^\ell \quad (2)$$

and

$$f_\gamma(x) = \left(\frac{m}{\bar{\gamma}}\right)^m \frac{x^{m-1}}{(m-1)!} e^{-m\frac{x}{\bar{\gamma}}}, \quad (3)$$

where $\bar{\gamma}$ is the average SNR per symbol on each diversity branch.

[‡]Our analytical results are applicable to any modulation format whose conditional BEP fits into the general expression (5).

Table I. Parameters for several noncoherent and differentially coherent modulation formats.

	Orthogonal binary signals	Nonorthogonal binary signals*	DPSK	DQPSK
a	0	$\left(\frac{1-\sqrt{1-\rho^2}}{2}\right)^{1/2}$	0	$\left(1-\frac{1}{\sqrt{2}}\right)^{1/2}$
b	1	$\left(\frac{1-\sqrt{1-\rho^2}}{2}\right)^{1/2}$	$\sqrt{2}$	$\left(1-\frac{1}{\sqrt{2}}\right)^{1/2}$
η	1	1	1	1

DPSK, differential phase-shift keying; DQPSK, differential quadrature phase-shift keying.

*Where $0 \leq \rho \leq 1$ is the magnitude of the cross-correlation coefficient between the two signals.

3. AVERAGE BIT ERROR PROBABILITY

3.1. General and exact analysis

Given the conditional BEP $P_b(x) \triangleq \Pr\{\text{bit error} | \gamma_S = x\}$ and the PDF at the output of the combiner f_{γ_S} , the average BEP for multibranch SEC is calculated by

$$\bar{P}_b = \int_0^{\infty} P_b(x) f_{\gamma_S}(x) dx. \quad (4)$$

A generic expression for the conditional BEP of noncoherent and differentially coherent modulations is given by [3, chap. 8] [11, eq. (4B.21)]

$$P_b(x) = Q_1(a\sqrt{x}, b\sqrt{x}) - \frac{\eta}{1+\eta} e^{-\frac{a^2+b^2}{2}x} I_0(abx), \quad (5)$$

where I_0 is the zero-order first-kind modified Bessel function, Q_1 is the first-order Marcum Q function defined in [3, Sect. 4.2], and a , b , and η are modulation-dependent parameters. A number of special cases are of particular importance, and their parameters[§] are specified in Table I.

In some special cases, the conditional BEP takes a simple form. When $a = 0$, for example, see orthogonal binary signaling and differential phase-shift keying (DPSK) in Table I, expression (5) reduces to [3, eq. 4.45]

$$P_b(x) = \frac{1}{1+\eta} e^{-\frac{b^2}{2}x}. \quad (6)$$

Thus, the general approach that follows can be easily circumvented, and the average BEP calculations are relatively simple. A similar simplification occurs when $b = 0$. This fact justifies the assumption $ab \neq 0$ adopted in the subsequent analysis.[¶]

One approach to compute the average BEP in Equation (4) is to express Equation (5) by its alternative form as a single finite integral, and then the integration over γ_S can be obtained from its MGF, which leads to a single finite integral expression [5, eq. (39)]. Here, the average BEP is derived by following a different approach in order to arrive at an exact closed-form expression.

Substituting Equations (5) and (1) into Equation (4) and after some simple algebraic manipulations, the following generic expression for the average BEP is obtained:

$$\begin{aligned} \bar{P}_b = & \alpha_{L,m}(\gamma_T; \bar{\gamma}) \left[\mathcal{I}_1(a, b, \bar{\gamma}, m) - \frac{\eta}{1+\eta} \mathcal{I}_2(a, b, \bar{\gamma}, m) \right] \\ & - \alpha_{L-1,m}(\gamma_T; \bar{\gamma}) \left[\mathcal{J}_1(\gamma_T; a, b, \bar{\gamma}, m) - \frac{\eta}{1+\eta} \mathcal{J}_2(\gamma_T; a, b, \bar{\gamma}, m) \right], \end{aligned} \quad (7)$$

[§]Note that Equation (5) is defined in terms of the instantaneous SNR per symbol γ_S ; thus, DQPSK parameters slightly differ from those given in [11] or [3].

[¶]Nevertheless, our expressions are also valid for $a = 0$ or $b = 0$ if these cases are properly interpreted as limits.

where $\alpha_{L,m}(\gamma_T; \bar{\gamma})$ are known coefficients defined as

$$\alpha_{L,m}(\gamma_T; \bar{\gamma}) \triangleq \frac{\left(\frac{m}{\bar{\gamma}}\right)^m}{(m-1)!} \left[\frac{1 - [F_\gamma(\gamma_T)]^L}{1 - F_\gamma(\gamma_T)} \right], \tag{8}$$

with $F_\gamma(\gamma_T)$ given in Equation (2) and where $\mathcal{I}_1, \mathcal{I}_2, \mathcal{J}_1,$ and \mathcal{J}_2 are integrals defined as

$$\begin{cases} \mathcal{I}_1(a, b, \bar{\gamma}, m) \triangleq \int_0^\infty x^{m-1} e^{-\frac{m}{\bar{\gamma}}x} Q_1(a\sqrt{x}, b\sqrt{x}) dx \\ \mathcal{I}_2(a, b, \bar{\gamma}, m) \triangleq \int_0^\infty x^{m-1} e^{-\left[\frac{m}{\bar{\gamma}} + \frac{a^2+b^2}{2}\right]x} I_0(abx) dx \\ \mathcal{J}_1(\gamma_T; a, b, \bar{\gamma}, m) \triangleq \int_0^{\gamma_T} x^{m-1} e^{-\frac{m}{\bar{\gamma}}x} Q_1(a\sqrt{x}, b\sqrt{x}) dx \\ \mathcal{J}_2(\gamma_T; a, b, \bar{\gamma}, m) \triangleq \int_0^{\gamma_T} x^{m-1} e^{-\left[\frac{m}{\bar{\gamma}} + \frac{a^2+b^2}{2}\right]x} I_0(abx) dx \end{cases} \tag{9}$$

What remains to achieve the goal of this analysis is to show that $\mathcal{I}_1, \mathcal{I}_2, \mathcal{J}_1,$ and \mathcal{J}_2 can be given in exact closed form by a finite combination of Marcum Q , Bessel, and elementary functions.

The complete integrals \mathcal{I}_1 and \mathcal{I}_2 are crucial for performance analysis of differentially coherent and noncoherent modulations in fading channels and were studied in [12, 13]. In [13], an exact closed-form expression for \mathcal{I}_1 was derived in terms of the Gauss hypergeometric function:

$$\begin{aligned} \mathcal{I}_1(a, b, \bar{\gamma}, m) = \frac{(m-1)!}{(m/\bar{\gamma})^m} & \left\{ 1 + \frac{b^2}{c_1} \sum_{\ell=0}^{m-1} (\ell+1) \left(\frac{2m}{\bar{\gamma}c_1}\right)^\ell \right. \\ & \times \left[\frac{a^2}{c_1} {}_2F_1\left(\frac{\ell+2}{2}, \frac{\ell+2}{2} + \frac{1}{2}; 2; \frac{4a^2b^2}{c_1^2}\right) \right. \\ & \left. \left. - \frac{1}{1+\ell} {}_2F_1\left(\frac{\ell+1}{2}, \frac{\ell+1}{2} + \frac{1}{2}; 1; \frac{4a^2b^2}{c_1^2}\right) \right] \right\}, \end{aligned} \tag{10}$$

where $c_1 \triangleq a^2 + b^2 + 2(m/\bar{\gamma})$. Applying [14, eq. 15.4.10] and the recurrence relations [10, eq. 8.731-4] and [10, eq. 8.914-1], the following identities are obtained:

$${}_2F_1\left(\frac{\ell+2}{2}, \frac{\ell+2}{2} + \frac{1}{2}; 2; c_2\right) = \frac{2c_2^{-1}(1-c_2)^{-\frac{\ell}{2}}}{2\ell+1} \left\{ P_{\ell+1}\left(\frac{1}{\sqrt{1-c_2}}\right) - P_{\ell-1}\left(\frac{1}{\sqrt{1-c_2}}\right) \right\}, \tag{11}$$

$${}_2F_1\left(\frac{\ell+1}{2}, \frac{\ell+1}{2} + \frac{1}{2}; 1; c_2\right) = \frac{(1-c_2)^{-\frac{\ell}{2}}}{2\ell+1} \left\{ (\ell+1)P_{\ell+1}\left(\frac{1}{\sqrt{1-c_2}}\right) + \ell P_{\ell-1}\left(\frac{1}{\sqrt{1-c_2}}\right) \right\}, \tag{12}$$

where $c_2 \triangleq 4a^2b^2/c_1^2$ and $P_n(\cdot)$ are the Legendre polynomials. Substituting Equations (11) and (12) into Equation (10) and after some algebra, we obtain the following expression for \mathcal{I}_1 in terms of elementary functions:

$$\begin{aligned} \mathcal{I}_1(a, b, \bar{\gamma}, m) &= (m-1)! \left(\frac{\bar{\gamma}}{m}\right)^m \\ &\times \left\{ 1 + \frac{b^2}{c_1} \sum_{\ell=0}^{m-1} \frac{\binom{\ell+1}{2\ell+1} \left(\frac{2m}{\bar{\gamma}c_1\sqrt{1-c_2}}\right)^\ell}{\left(\frac{c_1}{2b^2} - 1\right)^{-1}} \left[P_{\ell+1}\left(\frac{1}{\sqrt{1-c_2}}\right) \right. \right. \\ &\quad \left. \left. - \left(1 + \frac{\left(\frac{c_1}{2b^2} - 1\right)^{-1}}{\binom{\ell+1}{2\ell+1}} \right) P_{\ell-1}\left(\frac{1}{\sqrt{1-c_2}}\right) \right] \right\}. \end{aligned} \quad (13)$$

After a simple rescaling, a generalization of \mathcal{I}_2 is directly found in the table of integrals [10, eq. 6.624-5] in terms of the associate Legendre function. As in the case of \mathcal{I}_1 , \mathcal{I}_2 reduces to a simple expression in terms of Legendre polynomials:

$$\mathcal{I}_2(a, b, \bar{\gamma}, m) = \frac{(m-1)!}{(ab)^m} \left(\frac{c_2}{1-c_2}\right)^{\frac{m}{2}} P_{m-1}\left(\frac{1}{\sqrt{1-c_2}}\right). \quad (14)$$

Integrals \mathcal{J}_1 and \mathcal{J}_2 can be studied within the theory of the incomplete cylindrical functions developed by Agrest and Maksimov [15]. For convenience, we introduce the following incomplete Lipschitz–Hankel integrals (ILHI) of the k th order first-kind modified Bessel functions I_k :

$$I_{e,r,k}(x; \alpha) \triangleq \int_0^x t^r e^{-\alpha t} I_k(t) dt, \quad (15)$$

where $\alpha \in \mathbb{R}$, $r, k \in \mathbb{N}$; $\alpha > 1$ and $x \in [0, \infty)$. Integrals of type ILHI play a central role in the theory developed in [15], and recently, some new results have appeared in the literature concerning the particular family of integrals defined in Equation (15) (see [9]). To obtain exact closed-form expressions for \mathcal{J}_1 and \mathcal{J}_2 , we start connecting these integrals with the ILHI defined in Equation (15). On the one hand, let us consider Lemma 1 of [9].

Lemma 1

The following identity holds:

$$\begin{aligned} \int_0^x t^m e^{-\beta t} Q_n(a\sqrt{t}, b\sqrt{t}) dt &= \frac{m!}{\beta^{m+1}} \left\{ 1 - e^{-\beta x} Q_n(a\sqrt{x}, b\sqrt{x}) \sum_{r=0}^m \frac{\beta^r}{r!} x^r \right. \\ &\quad \left. + \frac{1}{2} \sum_{r=0}^m \frac{\beta^r}{r!} \frac{b^{n-r-1}}{a^{n+r}} (a I_{e,r,n}(abx; \alpha) - b I_{e,r,n-1}(abx; \alpha)) \right\}, \end{aligned} \quad (16)$$

where $\beta, a, b \in \mathbb{R}$, $m, n \in \mathbb{N}$; $\beta, a, b > 0$, $n \geq 1$, $x \in [0, \infty)$ and $\alpha = (a^2 + b^2 + 2\beta)/2ab > 1$.

After a simple rescaling and further simplifications, Lemma 1 is exploited to obtain the following expression for \mathcal{J}_1 :

$$\begin{aligned} \mathcal{J}_1(\gamma_T; a, b, \bar{\gamma}, m) &= (m-1)! \left(\frac{\bar{\gamma}}{m}\right)^m \left\{ 1 - e^{-\frac{m}{\bar{\gamma}}\gamma_T} Q_1(a\sqrt{\gamma_T}, b\sqrt{\gamma_T}) \sum_{\ell=0}^{m-1} \frac{\left(\frac{m}{\bar{\gamma}}\right)^\ell}{\ell!} \gamma_T^\ell \right. \\ &\quad + \frac{1}{2} \sum_{\ell=0}^{m-1} \frac{\left(\frac{m}{\bar{\gamma}}\right)^\ell}{\ell!} \frac{1}{(ab)^\ell} \left(I_{e,\ell,1}\left(ab\gamma_T; \frac{1}{\sqrt{c_2}}\right) \right. \\ &\quad \left. \left. - \frac{b}{a} I_{e,\ell,0}\left(ab\gamma_T; \frac{1}{\sqrt{c_2}}\right) \right) \right\}. \end{aligned} \quad (17)$$

On the other hand, a simple rescaling allows us to express \mathcal{J}_2 as

$$\mathcal{J}_2(\gamma_T; a, b, \bar{\gamma}, m) = \frac{1}{(ab)^m} I_{e_{m-1,0}} \left(ab\gamma_T; \frac{1}{\sqrt{c_2}} \right). \tag{18}$$

Finally, once \mathcal{J}_1 and \mathcal{J}_2 are expressed in terms of the ILHI defined in Equation (15), we apply Proposition 1 of [9] to express such ILHI as a finite combination of Marcum Q , Bessel, and elementary functions:

$$I_{e_{r,k}}(x; \alpha) = \mathcal{A}_{r,k}^0(\alpha) + \mathcal{A}_{r,k}^1(\alpha) Q_1 \left(\frac{\sqrt{x}}{\sqrt{\alpha + \sqrt{\alpha^2 - 1}}}, \sqrt{x} \sqrt{\alpha + \sqrt{\alpha^2 - 1}} \right) + e^{-\alpha x} \sum_{i=0}^r \sum_{j=0}^{k+1} \mathcal{B}_{r,k}^{i,j}(\alpha) x^i I_j(x), \tag{19}$$

where the set of coefficients $\{\mathcal{A}_{r,k}^l(\alpha), \mathcal{B}_{r,k}^{i,j}(\alpha)\}$ can be obtained recursively in a finite number of steps using the algorithm given in [9, Appendix III].[‡]

The final average BEP is obtained by substituting the derived expressions (13), (14), (17), and (18) for $\mathcal{I}_1, \mathcal{I}_2, \mathcal{J}_1$, and \mathcal{J}_2 , respectively, into Equation (7), which yields

$$\begin{aligned} \bar{P}_b = \alpha_{L,m}(\gamma_T; \bar{\gamma}) & \left[(m-1)! \left(\frac{\bar{\gamma}}{m} \right)^m \left\{ 1 + \left(\frac{1}{2} - \frac{b^2}{c_1} \right) \sum_{\ell=0}^{m-1} \binom{\ell+1}{2\ell+1} \left(\frac{2m}{\bar{\gamma} c_1 \sqrt{1-c_2}} \right)^\ell \right. \right. \\ & \times \left[P_{\ell+1} \left(\frac{1}{\sqrt{1-c_2}} \right) - \left(1 + \frac{1}{\left(\frac{\ell+1}{2\ell+1} \right) \left(\frac{c_1}{2b^2} - 1 \right)} \right) P_{\ell-1} \left(\frac{1}{\sqrt{1-c_2}} \right) \right] \Big\} \\ & - \frac{\eta}{1+\eta} \frac{(m-1)!}{(ab)^m} \left(\frac{c_2}{1-c_2} \right)^{\frac{m}{2}} P_{m-1} \left(\frac{1}{\sqrt{1-c_2}} \right) \Big] \\ & - \alpha_{L-1,m}(\gamma_T; \bar{\gamma}) \left[(m-1)! \left(\frac{\bar{\gamma}}{m} \right)^m \left\{ 1 - e^{-\frac{m}{\bar{\gamma}} \gamma_T} Q_1(a\sqrt{\gamma_T}, b\sqrt{\gamma_T}) \sum_{\ell=0}^{m-1} \frac{m^\ell}{\ell!} \right. \right. \\ & \times \left(\frac{\gamma_T}{\bar{\gamma}} \right)^\ell + \frac{1}{2} \sum_{\ell=0}^{m-1} \frac{m^\ell}{\ell!} \frac{1}{(ab\bar{\gamma})^\ell} \left(I_{e_{\ell,1}} \left(ab\gamma_T; \frac{1}{\sqrt{c_2}} \right) - \frac{b}{a} I_{e_{\ell,0}} \left(ab\gamma_T; \frac{1}{\sqrt{c_2}} \right) \right) \Big\} \\ & \left. - \frac{\eta}{1+\eta} \frac{1}{(ab)^m} I_{e_{m-1,0}} \left(ab\gamma_T; \frac{1}{\sqrt{c_2}} \right) \right], \tag{20} \end{aligned}$$

where the ILHI $I_{e_{r,k}}$ are represented in closed form by Equation (19) and the coefficients $\alpha_{L,m}, c_1$ and c_2 are as previously defined. Note that Equation (20) is an exact closed-form expression for the average BEP in terms of Bessel, Marcum Q , and elementary functions, which is general and valid for any modulation format whose conditional BEP can be expressed by Equation (5).

An interesting special case occurs when $\eta = 1$, which includes noncoherent detection of correlated binary signaling and DQPSK (see Table I). As noted in [3, Chap. 9], comparing equations (40) and (42) of [16], the following compact expression for the conditional BEP is obtained in this case ($\eta = 1$):

$$P_b(x) = \frac{1}{2} [1 - Q_1(b\sqrt{x}, a\sqrt{x}) + Q_1(a\sqrt{x}, b\sqrt{x})]. \tag{21}$$

[‡]MATLAB[™] and MATHEMATICA[™] programs to compute the set of coefficients $\{\mathcal{A}_{r,k}^l(\alpha), \mathcal{B}_{r,k}^{i,j}(\alpha)\}$, either numerically or symbolically, are found in the technical note posted in <http://webpersonal.uma.es/de/jfparis/>.

Then, after considering Equation (21) and $\eta = 1$, the average BEP in Equation (7) can be rewritten in terms of \mathcal{I}_1 and \mathcal{J}_1 as

$$\begin{aligned} \bar{P}_b &= \frac{1}{2} \alpha_{L,m}(\gamma_T; \bar{\gamma}) [1 - \mathcal{I}_1(b, a, \bar{\gamma}, m) + \mathcal{I}_1(a, b, \bar{\gamma}, m)] \\ &\quad - \frac{1}{2} \alpha_{L-1,m}(\gamma_T; \bar{\gamma}) [1 - \mathcal{J}_1(b, a, \bar{\gamma}, m) + \mathcal{J}_1(a, b, \bar{\gamma}, m)], \end{aligned} \quad (22)$$

where \mathcal{I}_1 and \mathcal{J}_1 are given in Equations (13) and (17), respectively. In this case, the average BEP is given in terms of two symmetric differences of \mathcal{I}_1 and \mathcal{J}_1 . For brevity reasons, the final expression is not explicitly written here.

3.2. Asymptotic analysis in the high signal-to-noise ratio regime

The derived average BEP expressions are given in terms of Bessel, Marcum Q , and elementary functions. Given that the analytical properties of these special functions are well studied, obtaining further insight from these expressions is straightforward, for example, upper and lower bounds or asymptotic approximations [3, chap. 4].

Moreover, the derived closed-form expressions for integrals \mathcal{I}_1 and \mathcal{I}_2 allow us to obtain an approximation of the average BEP in the high SNR regime. Let us consider $\bar{\gamma} \rightarrow \infty$ and, as a consequence, $\gamma_T \rightarrow \infty$. Thus, as the switching threshold increases, the involved incomplete integrals tend to their corresponding complete version, that is, $\mathcal{J}_1 \rightarrow \mathcal{I}_1$ and $\mathcal{J}_2 \rightarrow \mathcal{I}_2$. Taking this into account, the generic average BEP in Equation (7) can be approximated by

$$\bar{P}_b \approx \left(\frac{m}{\bar{\gamma}}\right)^m \frac{[F_\gamma(\gamma_T)]^{L-1}}{(m-1)!} \left[\mathcal{I}_1(a, b, \bar{\gamma}, m) - \frac{\eta}{1+\eta} \mathcal{I}_2(a, b, \bar{\gamma}, m) \right], \quad (23)$$

where F_γ is the cumulative distribution function of the instantaneous SNR per diversity branch, as previously defined. Then, \mathcal{I}_1 and \mathcal{I}_2 are replaced with Equations (13) and (14) into Equation (23), and further algebraic manipulations are performed to obtain the following approximation:

$$\begin{aligned} \bar{P}_b &\approx [F_\gamma(\gamma_T)]^{L-1} \left[\left\{ 1 + \left(\frac{1}{2} - \frac{b^2}{c_1}\right) \sum_{\ell=0}^{m-1} \left(\frac{\ell+1}{2\ell+1}\right) \left(\frac{2mc_3}{c_1\bar{\gamma}}\right)^\ell \right. \right. \\ &\quad \left. \left. \times \left[P_{\ell+1}(c_3) - \left(\frac{c_1(\ell+1) + \ell 2b^2}{(\ell+1)(c_1 - 2b^2)}\right) P_{\ell-1}(c_3) \right] \right\} - \frac{\eta}{1+\eta} \left(\frac{2mc_3}{c_1\bar{\gamma}}\right)^m P_{m-1}(c_3) \right], \end{aligned} \quad (24)$$

where for convenience we define $c_3 \triangleq 1/\sqrt{1-c_2}$ and c_2 is as previously defined. Note that the previous expression, which is given in terms of elementary functions, provides a good approximation of the average BEP in the high SNR regime, being asymptotically exact as $\bar{\gamma} \rightarrow \infty$.

4. NUMERICAL RESULTS

In this section, we show the usefulness of the obtained results in the analysis and design of multi-branch switched diversity systems with differentially coherent detection over Nakagami- m fading channels. Some numerical results are provided from the evaluation of the derived average BEP expressions. Besides, Monte Carlo simulations are presented in order to validate our analytical derivations.

Figure 1 depicts the average BEP as a function of the average SNR per branch $\bar{\gamma}$ for DQPSK (see Table I), considering different values of the Nakagami parameter m and number of branches L . The optimum switching threshold γ_T^* that minimizes the average BEP is adopted for every $\bar{\gamma}$. The optimum value γ_T^* has been obtained by applying standard numerical minimization methods to expression (20). The plotted curves show that the system performance within the 10^{-6} to 10^{-3} range is essentially determined by the product of m and L , which can be interpreted as a global diversity

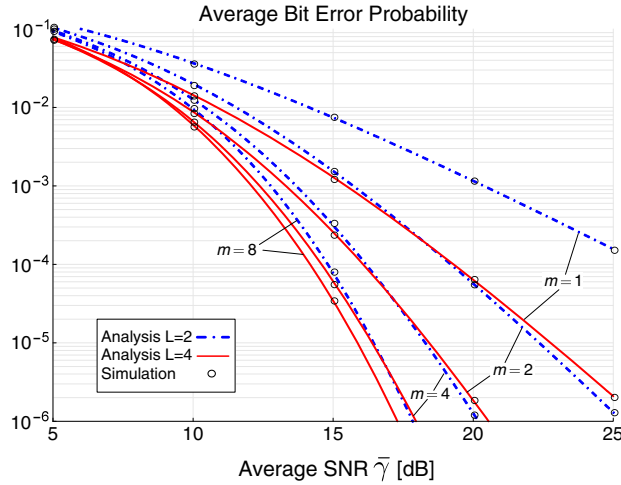


Figure 1. Average bit error probability versus average signal-to-noise ratio (SNR) per branch for differential quadrature phase-shift keying using optimum switching thresholds.

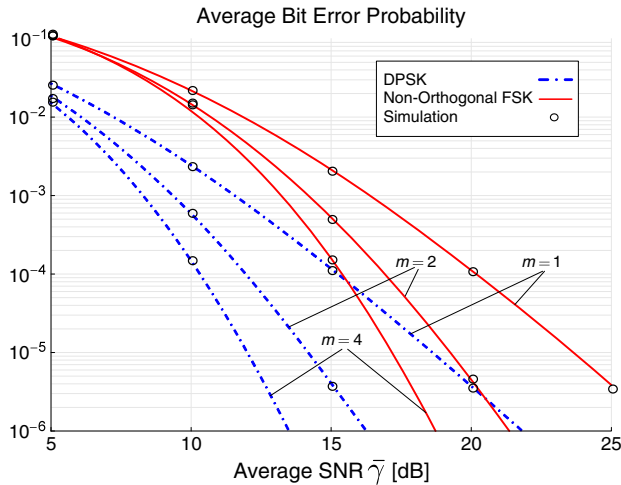


Figure 2. Average bit error probability for differential phase-shift keying (DPSK) and nonorthogonal frequency-shift keying (FSK; $\rho = 0.5$) modulation schemes with $L = 4$ diversity branches.

order measure. Simulation results are also superimposed to the analytical curves, confirming the validity of the derived expressions.

To illustrate the generality of the derived analytical results for different signaling formats (see Table I), Figure 2 shows the average BEP for nonorthogonal FSK ($\rho = 0.5$), compared with the corresponding results for DPSK. As in Figure 1, optimum switching thresholds are assumed. Figure 3 shows the trade-off between the correlation (frequency separation) in nonorthogonal FSK and the number of diversity branches for different values of the Nakagami- m parameter. It is observed that the performance loss associated to a larger cross-correlation is higher as m increases.

Figure 4 shows the sensitivity of the average BEP for DQPSK to the selection of the switching threshold γ_T . Let us define the relative error as $\epsilon \triangleq \gamma_T / \gamma_T^*$. When $\epsilon = 1$, the optimum threshold is always chosen, whereas $\epsilon \neq 1$ represents a certain relative deviation from the optimum value. The plotted curves show that multibranch switched diversity exhibits high sensitivity to a wrong switching threshold when the channel presents high values of m , for example, when the radio channel has a strong line-of-sight component. Also, it is shown that the average BEP degradation due to

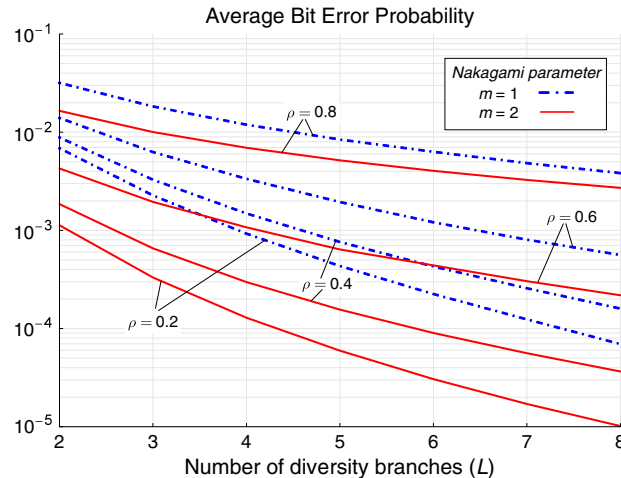


Figure 3. Average bit error probability versus number of diversity branches (L) with nonorthogonal binary frequency-shift keying signaling for different values of the cross-correlation (ρ) and the Nakagami- m parameter.

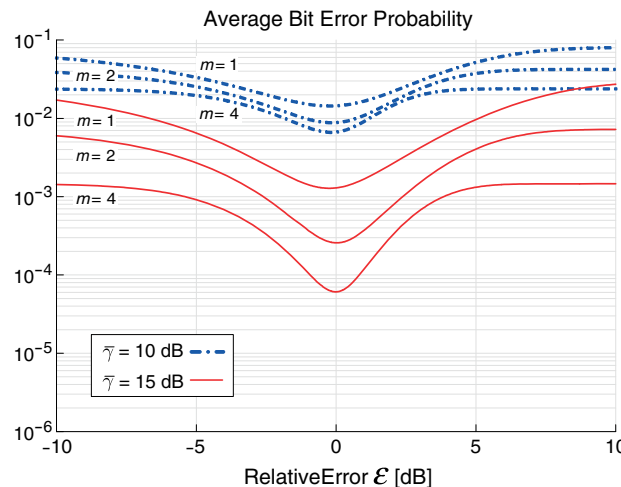


Figure 4. Average bit error probability versus the optimum threshold deviation for differential quadrature phase-shift keying with $L = 4$.

the threshold deviation is greater as the average SNR increases. For an average SNR of 15 dB and $m = 4$, the average BEP is degraded by one order of magnitude when $\epsilon = 2$ (3 dB).

Finally, in Figure 5 the asymptotic approximation of the average BEP for DQPSK in Equation (24) is compared with the exact closed-form expression in Equation (20). Both expressions have been numerically evaluated in the high SNR regime for several values of the number of branches L and the Nakagami parameter. It is shown that Equation (24) fits reasonably well with the exact average BEP in Equation (20) for average SNR values higher than 20 dB. Also, it can be seen that the approximation is asymptotically tighter as the average SNR increases.

5. CONCLUSIONS

In this paper, we have derived exact and closed-form expressions for the average BEP of noncoherent and differentially coherent detection in multibranch switched diversity systems under Nakagami- m fading. The derived expressions have led to easily computable results that are useful for the analysis and design of switched diversity based systems.

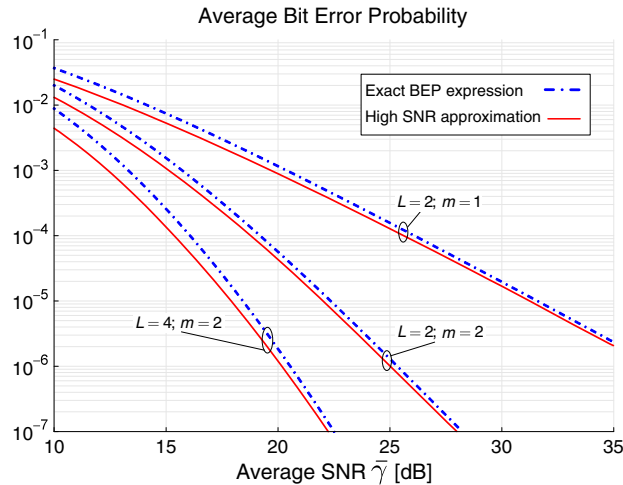


Figure 5. Exact average bit error probability (BEP) and high signal-to-noise ratio (SNR) approximation for differential quadrature phase-shift keying with optimum switching thresholds and several values of the number of branches L and Nakagami parameter m .

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