Closed-form analysis of dual-branch switched diversity with binary nonorthogonal signalling

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An exact closed-form analytical expression is derived for the average bit error probability of dual-branch switched diversity over Rayleigh fading. A practical scheme is considered employing binary nonorthogonal signalling with noncoherent detection. The analytical results are useful to evaluate the performance–bandwidth trade-off in systems that intentionally employ nonorthogonal signalling.

Introduction: Switched diversity has been thoroughly studied as an attempt at simplifying practical systems exploiting diversity. The simplest and best studied switched diversity system is the dual-branch switch-and-stay combining (SSC) over independent and identically distributed (i.i.d.) branches [1, Chap. 9]. Noncoherent detection of binary signals is frequently adopted in practical SSC systems as one of the least complex modulation schemes. In such a case, signals can be chosen nonorthogonal at the transmitter in order to reduce bandwidth utilisation, at the expense of certain performance degradation [2, Chap. 5].

Considerable attention has been paid to the performance analysis of SSC over i.i.d. fading channels [1, 3–5]. By following the moment generating function (MGF) approach, results in [1] and [3] for the average bit error probability (BEP) are in the form of single finite integrals. In [4], a new analysis is performed to give a closed-form BEP expression for coherent detection. An exact formula for noncoherent detection with orthogonal signalling is provided in [5]. However, to the best of the authors' knowledge, exact closed-form expressions for noncoherent detection of correlated signals are not found in the literature.

In this Letter, a new closed-form BEP analysis is presented for noncoherent detection of correlated binary signals with dual-branch switched diversity over i.i.d. Rayleigh fading. The resultant average BEP expression is in the form of Marcum Q and elementary functions, thus avoiding the need for numerical integration.

Average bit error probability: Let us assume a dual-branch SSC system under i.i.d. Rayleigh fading. The probability density function (pdf) of the instantaneous signal-to-noise ratio (SNR) per symbol γ_S at the output of the combiner is [1, eqn. (9.275)]

$$f_{\gamma_S}(x) = \begin{cases} \frac{1}{\bar{\gamma}}\left[1-\exp\left(-\frac{\gamma_T}{\bar{\gamma}}\right)\right]\exp\left(-\frac{x}{\bar{\gamma}}\right), & x < \gamma_T \\ \frac{1}{\bar{\gamma}}\left[2-\exp\left(-\frac{\gamma_T}{\bar{\gamma}}\right)\right]\exp\left(-\frac{x}{\bar{\gamma}}\right), & x \geq \gamma_T \end{cases} \tag{1}$$

where $\bar{\gamma}$ is the average SNR on each diversity branch and γ_T is the switching threshold. Given the conditional BEP $P_b(x) = \Pr\{\text{bit error}|\gamma_S = x\}$ the average BEP is calculated by

$$\bar{P}_b = \int_0^\infty P_b(x) f_{\gamma_S}(x)\,dx \tag{2}$$

After considering [6, eqns. (40) and (42)], the conditional BEP for noncoherent binary signalling given in [1, eqn. (8.70)] can be expressed in terms of the symmetric difference of Marcum Q functions, i.e.

$$P_b(x) = \frac{1}{2}\left[1 - Q\left(b\sqrt{x}, a\sqrt{x}\right) + Q\left(a\sqrt{x}, b\sqrt{x}\right)\right] \tag{3}$$

with $a = ((1-\sqrt{1-\rho^2})/2)^{1/2}$ and $b = ((1+\sqrt{1-\rho^2})/2)^{1/2} = \rho/(2a)$, where ρ is the magnitude of the cross-correlation between the two signals.

Substituting (1) and (3) into (2) and after some algebraic manipulations the following expression for the average BEP is obtained:

$$\bar{P}_b = \frac{1}{2\bar{\gamma}}\left(2 - e^{-\frac{\gamma_T}{\bar{\gamma}}}\right)[\bar{\gamma} - U(b,a,\bar{\gamma}) + U(a,b,\bar{\gamma})] - \frac{1}{2\bar{\gamma}}\left[\bar{\gamma}\left(1 - e^{-\frac{\gamma_T}{\bar{\gamma}}}\right) - V(b,a,\bar{\gamma}) + V(a,b,\bar{\gamma})\right] \tag{4}$$

where U and V are integrals defined as

$$\begin{cases} U(a,b,\bar{\gamma}) \doteq \int_0^\infty e^{-\frac{x}{\bar{\gamma}}} Q(a\sqrt{x}, b\sqrt{x})\,dx \\ V(\gamma_T; a,b,\bar{\gamma}) \doteq \int_0^{\gamma_T} e^{-\frac{x}{\bar{\gamma}}} Q(a\sqrt{x}, b\sqrt{x})\,dx \end{cases} \tag{5}$$

What remains to complete the analysis is to show that U and V may be given in exact closed-form by a finite combination of Marcum Q and elementary functions.

The complete integral U is crucial for performance analysis of noncoherent modulations in fading channels. In [7], an exact closed-form expression for U was derived in terms of Gauss hypergeometric functions, which can be easily expressed in terms of Legendre polynomials. Then, after some algebra the following symmetric difference of U functions is obtained:

$$U(a,b,\bar{\gamma}) - U(b,a,\bar{\gamma}) = \frac{\bar{\gamma}}{\sqrt{c^2-\rho^2}}(a^2-b^2) \tag{6}$$

where $c \doteq 1 + 2/\bar{\gamma}$.

On the other hand, the integral V can be studied within the theory of the incomplete cylindrical functions developed by Agrest and Maksimov [8]. Generic forms of the incomplete integral V have been recently investigated in [9]. After simple rescaling and further simplifications, lemma 1 of [9] can be exploited to obtain the following symmetric difference of V functions:

$$\begin{aligned} &V(a,b,\bar{\gamma}) - V(b,a,\bar{\gamma}) \\ &= \bar{\gamma}\left\{ e^{-\frac{\gamma_T}{\bar{\gamma}}}\left(Q(b\sqrt{\gamma_T}, a\sqrt{\gamma_T}) - Q(a\sqrt{\gamma_T}, b\sqrt{\gamma_T})\right) \right. \\ &\left. + \frac{1}{2}\left(\frac{a}{b} - \frac{b}{a}\right)\int_0^{ab\gamma_T} e^{\frac{-c}{\rho}t} I_0(t)\,dt \right\} \end{aligned} \tag{7}$$

where I_0 is the first-kind modified Bessel function. The integral involving I_0 in (7) is an incomplete Lipschitz-Hankel integral, which can be expressed in terms of Marcum Q, Bessel and elementary functions [9]. Hence, by taking into account lemma 3 of [9] and the connection between Bessel and Marcum Q functions pointed out in [6], the following compact expression is found for the integral in (7):

$$\begin{aligned} &\int_0^{ab\gamma_T} e^{\frac{-c}{\rho}t} I_0(t)dt \\ &= \frac{-\rho}{\sqrt{c^2-\rho^2}}\left[Q\left(g\sqrt{\gamma_T}, \frac{\rho}{2g}\sqrt{\gamma_T}\right) - Q\left(\frac{\rho}{2g}\sqrt{\gamma_T}, g\sqrt{\gamma_T}\right)\right] \end{aligned} \tag{8}$$

where $g(\bar{\gamma}) \doteq \rho/\left(\sqrt{2}\sqrt{c+\sqrt{c^2-\rho^2}}\right)$. Then, after substituting (8) into (7), the two symmetric differences in (6) and (7) can be replaced in (4) to obtain the final average BEP expression:

$$\begin{aligned} \bar{P}_b(\bar{\gamma};\rho,\gamma_T) = \frac{1}{2}\Bigg[1 + \left(e^{-\frac{\gamma_T}{\bar{\gamma}}} - 2\right) f(\bar{\gamma}) + e^{\frac{\gamma_T}{\bar{\gamma}}} \\ \times \left[Q(a\sqrt{\gamma_T}, b\sqrt{\gamma_T}) - Q(b\sqrt{\gamma_T}, a\sqrt{\gamma_T})\right] \\ - f(\bar{\gamma})\left[Q\left(g(\bar{\gamma})\sqrt{\gamma_T}, \frac{\rho}{2g(\bar{\gamma})}\sqrt{\gamma_T}\right)\right. \\ \left. - Q\left(\frac{\rho}{2g(\bar{\gamma})}\sqrt{\gamma_T}, g(\bar{\gamma})\sqrt{\gamma_T}\right)\right]\Bigg] \end{aligned} \tag{9}$$

with $f(\bar{\gamma}) \doteq \sqrt{1-\rho^2}/\sqrt{c^2-\rho^2}$. Note that the previous expression is given in closed-form in terms of Marcum Q and elementary functions. Given that analytical properties of Marcum Q function are well-studied, obtaining further insight from (9) is straightforward, e.g. simple upper bounds or asymptotic approximations [1, Sect. 4.2]. Moreover, the presented analytical approach is directly applicable to any modulation with conditional BEP in the form given in (3) (e.g. differentially coherent modulation such as DQPSK).

Numerical results: The average BEP expression in (9) has been evaluated in order to show the performance–bandwidth trade-off for correlated binary frequency shift keying (FSK) signals. The switching threshold that minimises the average BEP has been obtained by means of standard numerical minimisation techniques. Fig. 1 shows the optimum switching threshold against SNR for different values of the cross-correlation. By using the optimum switching threshold, average BEP against average SNR is plotted in Fig. 2 for different values of the cross-correlation parameter. In the same Figure, simulation results are superimposed on the analytical curves.

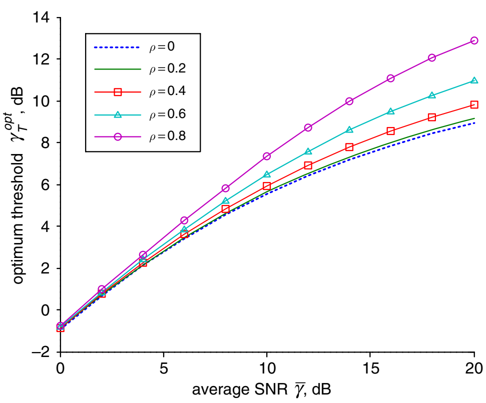

Fig. 1 *Optimum switching threshold against average SNR for different values of cross-correlation parameter*

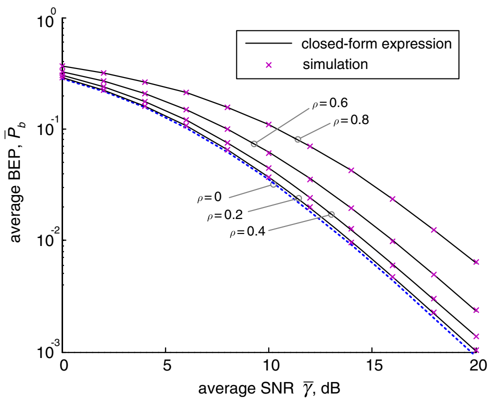

Fig. 2 *Average BEP against average SNR for different values of cross-correlation parameter (optimum adaptive switching threshold is used)*

Note that for two correlated FSK signals with $\rho = 0.4$, a 35% reduction of frequency separation is achieved (see [2, eqn. (5.137)]), with the corresponding bandwidth utilisation saving. In such a case, a performance loss of only 1 dB is experienced with respect to the orthogonal case ($\rho = 0$) for a typical 10 dB average SNR (see Fig. 2).

Conclusions: A simple and elegant closed-form expression for the average BEP of noncoherent and nonorthogonal binary signalling with switched diversity is derived in terms of the symmetric difference of Marcum Q functions. This expression leads to easily computable results, which are useful for the design of switched diversity based systems.

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