

# Outage Probability Analysis for Nakagami- $q$ (Hoyt) Fading Channels under Rayleigh Interference

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**Abstract**—Exact closed-form expressions are obtained for the outage probability of Nakagami- $q$  (Hoyt) fading channels under co-channel interference (CCI). The scenario considered in this work assumes the joint presence of background white Gaussian noise and independent Rayleigh interferers with arbitrary powers.

**Index Terms**—Outage probability, Nakagami- $q$  (Hoyt) fading, co-channel interference (CCI).

## I. INTRODUCTION

OUTAGE probability is a key performance metric of wireless communication systems under co-channel interference (CCI). An excellent explanation of this topic can be found in [1, chapter 10] and references therein. The Nakagami- $q$  distribution, also referred to as Hoyt distribution [2], is commonly used to describe the short-term signal variation of certain wireless communication systems subject to fading [1]-[3]. Specifically, the Hoyt channel model has been applied in satellite-based cellular communications to characterize more severe fading conditions than those modeled by Rayleigh [3]-[4]. Although considerable attention has been paid to outage probability analysis, few published results for Hoyt fading channels are found in the literature, mainly due to reasons of mathematical tractability<sup>1</sup>. Recently, exact closed-form results for the outage probability of *interference-free* Hoyt fading channels were published in [5]. Moreover, only a few works in the literature include background noise in the outage probability analysis. The analysis in [6] includes background noise and assumes Nakagami- $n$  (Rician) or Nakagami- $m$  fading for the desired signal and Rayleigh faded interferers. Recently, closed-form expressions were provided in [4] for the outage probability of Rayleigh fading under mixed Rayleigh and Hoyt interference. Also, the same scenario which is considered in this work, i.e., a Hoyt faded signal with Rayleigh interferers, was analyzed in [4]. However, in the latter case the outage probability was given in the form of an infinite series expansion.

In this paper, closed-form expressions for the outage probability of Hoyt fading channels are derived, generalizing [5] by assuming the joint presence of background noise and

independent Rayleigh interferers with arbitrary powers. These results are obtained through an appropriate generalization of the moment-generating function (MGF) of the Hoyt fading distribution.

The remainder of this paper is organized as follows. The general problem formulation is presented in Section II. The generalization of the MGF is obtained in Section III and the outage probability expressions are derived in Section IV. Finally, some numerical results are given in Section V and conclusions are presented in Section VI.

## II. GENERAL PROBLEM FORMULATION

Let us assume the signal from the desired user at the receive antenna to be affected by Hoyt fading, while co-channel interference (CCI) signals are assumed to experience independent Rayleigh fading. The power  $X$  of the desired signal follows a Hoyt distribution with mean  $W_s$  and parameter  $q$ . The probability density function (PDF) of  $X$  is given by [1, eq.(2.11)]

$$f_X(x) = \frac{(1+q^2)}{2q\Omega_X} \exp\left(-\frac{(1+q^2)}{4q^2} \frac{x}{\Omega_X}\right) I_0\left(\frac{(1-q^4)}{4q^2} \frac{x}{\Omega_X}\right), \quad (1)$$

where  $I_0$  is the zeroth order modified Bessel function of the first kind, the parameter  $\Omega_X = W_s$  is the mean of  $X$ , and  $q$  is the Nakagami- $q$  fading parameter which ranges from 0 to 1. Both extremes of the range of  $q$  are interpreted as a limit. Let us divide the total number of interferers  $L$  into  $J$  groups, where every interferer in a group has the same mean power  $W_i$ . Consider  $n_i$  interferers in a given group with mean power  $W_i$ . It is shown in [6] that the outage probability in this scenario can be computed as

$$P_{out} \doteq \Pr\left\{\frac{X}{Y+\sigma^2} \leq \eta\right\} = \underbrace{\int_0^{\eta\sigma^2} f_X(x) dx}_{P_{out}^*} + \sum_{i=1}^J \sum_{j=1}^{n_i} \sum_{k=0}^{n_i-j} \sum_{l=0}^k E_{i,j} \frac{e^{\sigma^2/W_i} (-\sigma^2)^{k-l}}{l!(k-l)!W_i^k \eta^l} \int_{\eta\sigma^2}^{\infty} x^l e^{-\frac{x}{\eta W_i}} f_X(x) dx, \quad (2)$$

where  $Y$  is the power of the interfering signals,  $\eta$  is a predefined threshold,  $\sigma^2$  is the background noise power,  $E_{i,j}$  are certain constants defined in [6, eq. 6], and  $f_X$  is the PDF of the Hoyt fading distribution given in (1).

The first term in (2) represents the outage probability in the interference-free case. Recently, a simple exact closed-form expression was obtained for this term<sup>2</sup> [5]

$$P_{out}^* = Q\left(u\sqrt{\frac{\eta\sigma^2}{W_s}}, v\sqrt{\frac{\eta\sigma^2}{W_s}}\right) - Q\left(v\sqrt{\frac{\eta\sigma^2}{W_s}}, u\sqrt{\frac{\eta\sigma^2}{W_s}}\right), \quad (3)$$

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<sup>1</sup>In particular, even if the background noise is neglected, the general approach adopted in [1, chapter 10] is not applicable since the Gaussian characterization of the Hoyt distribution is not circularly symmetric.

where  $u(q) \doteq \frac{\sqrt{1-q^4}}{2q} \sqrt{\frac{1+q}{1-q}}$ ,  $v(q) \doteq \frac{\sqrt{1-q^4}}{2q} \sqrt{\frac{1-q}{1+q}}$  and  $Q$  is the Marcum Q function.

The second term in (2) represents the impact of the interference on the outage probability. Since this term consists of a finite number of generalized MGFs, the next section is devoted to computing this statistical function for the Hoyt fading distribution.

### III. INCOMPLETE GENERALIZED MGF OF THE HOYT FADING DISTRIBUTION

Let us focus on the following generalization of the MGF.

*Definition 1 (Incomplete Generalized MGF):* Let us consider a continuous random variable (RV)  $X$  with PDF  $f_X(x)$ . The Incomplete Generalized MGF (IG-MGF) of  $X$ , if it exists, is defined as

$$\mathcal{G}_X(n, s; \zeta) = \int_{\zeta}^{\infty} x^n e^{sx} f_X(x) dx,$$

where<sup>3</sup>  $s \in \mathbb{C}$ ,  $n$  is a nonnegative integer and  $\zeta \in \mathbb{R}$  with  $\zeta \geq 0$ .

Note that Definition 1 includes, as particular cases, several important statistical functions associated to the RV  $X$ :  $\mathcal{G}_X(0, 0; \zeta)$  is the complementary cumulative distribution function (CDF);  $\mathcal{G}_X(0, s; 0)$  is the MGF;  $\mathcal{G}_X(0, s; \zeta)$  is the marginal MGF and  $\mathcal{G}_X(n, s; 0)$  is the generalized MGF. To facilitate the analysis the following concepts are introduced.

*Definition 2 (Complementary IG-MGF):* The complementary IG-MGF  $\tilde{\mathcal{G}}_X(n, s; \zeta)$  of a RV  $X$  is (if it exists)  $\tilde{\mathcal{G}}_X(n, s; \zeta) = \int_0^{\zeta} x^n e^{sx} f_X(x) dx$ .

*Definition 3 (Normalized Hoyt Distributed RV):* For a given Hoyt distributed RV  $X$  with mean  $E[X] = \Omega_X$  and Hoyt parameter  $q$ , we define a 'normalized' Hoyt RV as  $\langle X \rangle \doteq \frac{1-q^4}{4q^2\Omega_X} X$ .

Some useful properties of the Hoyt distribution are summarized in the following Lemma.

*Lemma 1:* Let us consider a Hoyt distributed RV  $X$  with mean  $E[X] = \Omega_X$  and Hoyt parameter  $q$ . Sufficient conditions for the existence of the statistical functions  $\mathcal{G}_X(n, s; \zeta)$ ,  $\tilde{\mathcal{G}}_X(n, s; \zeta)$ ,  $\mathcal{G}_{\langle X \rangle}(n, s; \zeta)$  and  $\tilde{\mathcal{G}}_{\langle X \rangle}(n, s; \zeta)$  are:  $n \geq 0$  and  $\Re\{s\} < \frac{2q^2}{1-q^2}$ . In such a case, the following equalities hold

$$\begin{cases} \mathcal{G}_X(n, s; \zeta) = \left( \frac{4q^2\Omega_X}{1-q^4} \right)^n \mathcal{G}_{\langle X \rangle} \left( n, \frac{4sq^2\Omega_X}{1-q^4}; \frac{1-q^4}{4q^2\Omega_X} \zeta \right) \\ \tilde{\mathcal{G}}_{\langle X \rangle}(n, s; \zeta) + \mathcal{G}_{\langle X \rangle}(n, s; \zeta) = \frac{2q(n!)}{1-q^2} \bar{\alpha}(s)^{n+1} P_n(\alpha(s)\bar{\alpha}(s)) \end{cases}, \quad (4)$$

where  $\alpha(s) \doteq \frac{1+q^2}{1-q^2} - s$ ,  $\bar{\alpha}(s) \doteq \frac{1}{\sqrt{\alpha(s)^2-1}}$  and  $P_n$  is the Legendre polynomial of degree  $n$ .

*Proof:* See Appendix A. ■

Now, we express  $\tilde{\mathcal{G}}_{\langle X \rangle}(n, s; \zeta)$  in closed-form by the following Proposition.

*Proposition 1:* Let us consider a Hoyt distributed RV  $X$  with  $E[X] = \Omega_X$  and Hoyt parameter  $q$ . Then, if  $n \geq 0$  and

<sup>2</sup>A minor error in [5, eq. 9] is corrected here.

<sup>3</sup>The variable  $s$  is only evaluated along the real line, however the complex domain is assumed here, in accordance with the usual definition of the MGF in the context of communications theory [1].

$\Re\{s\} < \frac{2q^2}{1-q^2}$ , the complementary IG-MGF of  $\langle X \rangle$  is given by

$$\begin{aligned} \tilde{\mathcal{G}}_{\langle X \rangle}(n, s; \zeta) &= \frac{2q}{1-q^2} \left\{ A_n(s) \right. \\ &+ B_n(s) Q \left( \frac{\sqrt{\zeta}}{\sqrt{\alpha(s)+\bar{\alpha}(s)^{-1}}}, \sqrt{\zeta} \sqrt{\alpha(s)+\bar{\alpha}(s)^{-1}} \right) \\ &\left. + e^{-\alpha(s)\zeta} \sum_{\ell=0}^n [C_n^\ell(s) I_0(\zeta) + D_n^\ell(s) I_1(\zeta)] \zeta^\ell \right\}, \end{aligned} \quad (5)$$

where  $Q$  is the Marcum Q function,  $I_1$  is the first order modified Bessel function and,  $A_n(s)$ ,  $B_n(s)$ ,  $C_n^\ell(s)$  and  $D_n^\ell(s)$  are obtained recursively in a finite number of steps as follows:

$$\begin{cases} \underline{n=0} \begin{cases} A_0(s) = \bar{\alpha}, \\ B_0(s) = -2\bar{\alpha}, \\ C_0^0(s) = \bar{\alpha}, \\ D_0^0(s) = 0, \end{cases} & \underline{n=1} \begin{cases} A_1(s) = \alpha\bar{\alpha}^3, \\ B_1(s) = -2\alpha\bar{\alpha}^3, \\ C_1^0(s) = \alpha\bar{\alpha}^3, \\ C_1^1(s) = -\alpha\bar{\alpha}^2, \\ D_1^0(s) = 0, \\ D_1^1(s) = -\bar{\alpha}^2, \end{cases} \end{cases}$$

$$\underline{n \geq 2}$$

$$A_n(s) = (2n-1)\alpha\bar{\alpha}^2 A_{n-1} - (n-1)^2 \bar{\alpha}^2 A_{n-2},$$

$$B_n(s) = (2n-1)\alpha\bar{\alpha}^2 B_{n-1} - (n-1)^2 \bar{\alpha}^2 B_{n-2},$$

$$C_n^\ell(s) = \begin{cases} (2n-1)\alpha\bar{\alpha}^2 C_{n-1}^\ell - (n-1)^2 \bar{\alpha}^2 C_{n-2}^\ell & \text{for } 0 \leq \ell = n-2, \\ (2n-1)\alpha\bar{\alpha}^2 C_{n-1}^{n-1} + (n-1)\bar{\alpha}^2 & \text{for } \ell = n-1, \\ -\alpha\bar{\alpha}^2 & \text{for } \ell = n, \end{cases}$$

$$D_n^\ell(s) = \begin{cases} (2n-1)\alpha\bar{\alpha}^2 D_{n-1}^\ell - (n-1)^2 \bar{\alpha}^2 D_{n-2}^\ell & \text{for } 0 \leq \ell = n-2, \\ (2n-1)\alpha\bar{\alpha}^2 D_{n-1}^{n-1} & \text{for } \ell = n-1, \\ -\bar{\alpha}^2 & \text{for } \ell = n. \end{cases} \quad (6)$$

The functions  $\alpha(s)$  and  $\bar{\alpha}(s)$  are defined as in Lemma 1.

*Proof:* See Appendix B. ■

Finally, by taking Proposition 1 and Lemma 1 into account, we obtain a closed-form expression for  $\mathcal{G}_X(n, s; \zeta)$ .

*Corollary 1:* Under the conditions of Proposition 1, the IG-MGF of the Hoyt distributed RV  $X$  is given by

$$\begin{aligned} \mathcal{G}_X(n, s; \zeta) &= \left( \frac{4q^2\Omega_X}{1-q^4} \right)^n \left\{ \frac{2q(n!)}{1-q^2} \bar{\alpha} \left( \frac{4sq^2\Omega_X}{1-q^4} \right)^{n+1} \right. \\ &P_n \left( \alpha \left( \frac{4sq^2\Omega_X}{1-q^4} \right) \bar{\alpha} \left( \frac{4sq^2\Omega_X}{1-q^4} \right) \right) \\ &\left. - \tilde{\mathcal{G}}_{\langle X \rangle} \left( n, \frac{4sq^2\Omega_X}{1-q^4}; \frac{1-q^4}{4q^2\Omega_X} \zeta \right) \right\}, \end{aligned} \quad (7)$$

where  $\tilde{\mathcal{G}}_{\langle X \rangle}(n, s; \zeta)$  is given in (5).

### IV. OUTAGE PROBABILITY RESULTS

The mathematical tools provided in the previous section are now used to obtain exact closed-form and easily computable expressions for the outage probability in Hoyt fading channels under Rayleigh interference. To the best of the authors' knowledge, the expressions derived below are novel.

### A. Arbitrary number of interfering signals

The general case assumes an arbitrary number of interferers  $L$ . Note that the integrals in the second term of (2) can be easily identified with the IG-MGF as

$$\int_{\eta\sigma^2}^{\infty} x^l e^{-\frac{x}{\eta W_i}} f_X(x) dx = \mathcal{G}_X\left(l, -\frac{1}{\eta W_i}; \eta\sigma^2\right).$$

Then, the final outage probability expression is obtained by substituting (3) into (2) and replacing the previous integral with the IG-MGF as

$$P_{out} = Q\left(u\sqrt{\frac{\eta\sigma^2}{W_s}}, v\sqrt{\frac{\eta\sigma^2}{W_s}}\right) - Q\left(v\sqrt{\frac{\eta\sigma^2}{W_s}}, u\sqrt{\frac{\eta\sigma^2}{W_s}}\right) + \sum_{i=1}^J \sum_{j=1}^{n_i} \sum_{k=0}^{n_i-j} \sum_{l=0}^k E_{i,j} \frac{e^{\sigma^2/W_i} (-\sigma^2)^{k-l}}{l!(k-l)! W_i^k \eta^l} \mathcal{G}_X\left(l, \frac{-1}{\eta W_i}; \eta\sigma^2\right), \quad (8)$$

where the coefficients  $E_{i,j}$  are given in [6, eq. 6] and the IG-MGF  $\mathcal{G}_X(\cdot, \cdot, \cdot)$  is directly obtained from Corollary 1. Note that the outage probability in (8) is expressed in terms of a finite combination of first order Marcum Q, Bessel, and elementary functions.

### B. One dominant interfering signal

This particular scenario assumes one dominant interferer with mean power  $W_1$ , either neglecting the remainder  $L-1$  of the interferers or including them in the analysis as background Gaussian noise. In this case, the outage probability expression can be significantly simplified. By setting  $J = n_1 = 1$  in (2) and considering the expression for  $\mathcal{G}_X\left(0, -\frac{1}{\eta W_1}; \eta\sigma^2\right)$ , which follows from Corollary 1, the outage probability is obtained as

$$P_{out} = P_{out}^* + \frac{2qe^{\sigma^2/W_1}}{1-q^2} \left\{ -\bar{\alpha}(s_1) e^{-\alpha(s_1)\zeta_0} I_0(\zeta_0) + 2\bar{\alpha}(s_1) Q\left(\frac{\sqrt{\zeta_0}}{\sqrt{\alpha(s_1)+\bar{\alpha}(s_1)^{-1}}}, \sqrt{\zeta_0}\sqrt{\alpha(s_1)+\bar{\alpha}(s_1)^{-1}}\right) \right\}, \quad (9)$$

where  $s_1 \doteq -\frac{W_s}{\eta W_1} \frac{4q^2}{1-q^4}$ ,  $\zeta_0 \doteq \frac{1-q^4}{4q^2} \frac{\eta\sigma^2}{W_s}$  and  $\alpha(\cdot)$ ,  $\bar{\alpha}(\cdot)$  are defined as in previous sections.

In order to obtain further insight from the outage probability expression in (9), the following well-known metrics are defined:

$$\begin{cases} \text{SNR} \doteq \frac{W_s}{\eta\sigma^2} \\ \text{SIR} \doteq \frac{W_s}{\eta W_1} \end{cases} \quad \begin{cases} \text{SINR} \doteq \frac{W_s}{\eta W_1 + \eta\sigma^2} \\ \text{INR} \doteq \frac{\eta W_1}{\eta\sigma^2} \end{cases}, \quad (10)$$

where SNR and SIR are the signal-to-noise and signal-to-interference ratios, while SINR and INR are the signal-to-interference-plus-noise and interference-to-noise ratios, respectively. Note that these metrics are defined as normalized and averaged magnitudes. The first pair of metrics can be expressed as a function of the second pair as

$$\begin{cases} \text{SNR} = (1 + \text{INR}) \text{SINR} \\ \text{SIR} = (1 + \text{INR}^{-1}) \text{SINR} \end{cases}. \quad (11)$$

Then, after substituting (3) into (9) and considering [1, eq. 9.107] as well as previous definitions, the outage probability can be written in closed-form as

$$P_{out} = Q\left(\frac{u}{\sqrt{\text{SNR}}}, \frac{v}{\sqrt{\text{SNR}}}\right) - Q\left(\frac{v}{\sqrt{\text{SNR}}}, \frac{u}{\sqrt{\text{SNR}}}\right) + \exp\left(\frac{\text{SIR}}{\text{SNR}}\right) \left(\frac{u^2 - v^2}{2uv}\right) \bar{\alpha}\left(\frac{-\text{SIR}}{uv}\right) \left[1 - Q\left(\frac{h(\text{SIR})}{\sqrt{\text{SNR}}}, \frac{f(\text{SIR})}{\sqrt{\text{SNR}}}\right) + Q\left(\frac{f(\text{SIR})}{\sqrt{\text{SNR}}}, \frac{h(\text{SIR})}{\sqrt{\text{SNR}}}\right)\right], \quad (12)$$

where

$$\begin{cases} h(\text{SIR}) \doteq \sqrt{uv} \sqrt{\alpha\left(\frac{-\text{SIR}}{uv}\right) + \bar{\alpha}\left(\frac{-\text{SIR}}{uv}\right)^{-1}} \\ f(\text{SIR}) \doteq \frac{uv}{h(\text{SIR})} \end{cases},$$

and  $\alpha(\cdot)$ ,  $\bar{\alpha}(\cdot)$ ,  $u$  and  $v$  are as previously defined. Note that (12) is expressed in terms of the well-known SIR and SNR metrics. Besides, this expression involves only elementary and Marcum Q functions. Given that the analytical properties of the Marcum Q function are well-studied, obtaining further insight from (12) is straightforward, e.g. upper bounds or asymptotic approximations [1, chapter 4]. As an example, the derivative of the Marcum Q function in [12, eq. 2] is exploited here to obtain a second order Taylor approximation of Q as follows

$$Q(a\sqrt{t}, b\sqrt{t}) \approx 1 - t \cdot \frac{b^2}{2} + t^2 \cdot \frac{1}{2} \left(\frac{a^2 b^2}{2} + \frac{b^4}{4}\right). \quad (13)$$

Hence, after considering (13) with  $t \doteq 1/\text{SNR}$  and  $t \rightarrow 0$ , the following approximation can be found

$$P_{out} \approx \frac{u^2 - v^2}{2 \cdot \text{SNR}} + \frac{v^4 - u^4}{8 \cdot \text{SNR}^2} + \exp\left(\frac{1}{\text{INR}}\right) \left(\frac{u^2 - v^2}{2uv}\right) \bar{\alpha}\left(\frac{-\text{SIR}}{uv}\right) \left[1 + \frac{f^2 - h^2}{2 \cdot \text{SNR}} + \frac{h^4 - f^4}{8 \cdot \text{SNR}^2}\right]. \quad (14)$$

Note that the previous expression provides a good approximation of the outage probability in the high SNR regime, being asymptotically tight as  $\text{SNR} \rightarrow \infty$ . For a given value of SINR, the tradeoff between interference and noise is represented by INR. When interference dominates over noise, i.e.  $\text{INR} \rightarrow \infty$ , then  $\text{SNR} \rightarrow \infty$ . Therefore, for a given SINR, the approximation in (14) fits well with the exact outage probability in the high INR regime.

On the other hand, when noise dominates over interference, i.e.,  $\text{INR} \rightarrow 0$ , then  $\text{SNR} \rightarrow \text{SINR}$  and  $\text{SIR} \rightarrow \infty$ . In this case the outage probability can be approximated by (3), which corresponds to the interference-free case.

## V. NUMERICAL RESULTS

Fig. 1 and Fig. 2 show some results from the general expression (8). Fig. 1 represents the outage probability related to the normalized average SINR expressed in decibels

$$10 \log_{10} \left(\frac{W_s}{\eta \sum W_i + \eta\sigma^2}\right),$$

for several values of the Hoyt parameter  $q$ . In this particular example three interferers are considered with  $W_1 = 1/4$  and

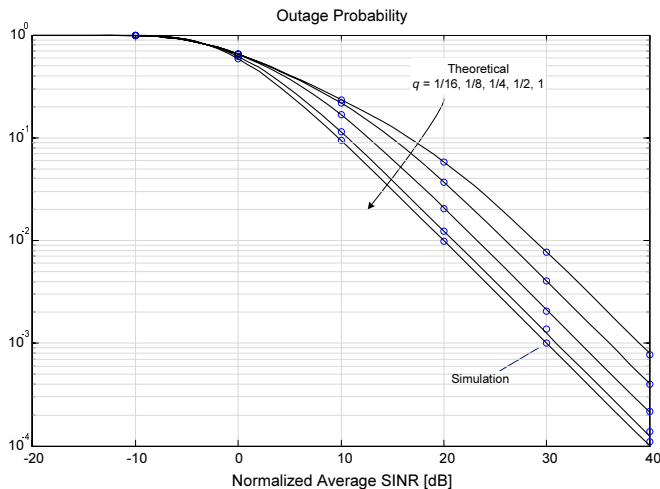


Fig. 1. Outage probability versus normalized average SINR. Three interferers are considered with  $W_1 = 1/4$ ,  $W_2 = W_3 = 1/8$ , and background noise power  $\sigma^2 = 1/10$ .

$W_2 = W_3 = 1/8$ , while the background noise power is  $\sigma^2 = 1/10$ . In Fig. 2 the outage probability versus the number of interferers is plotted for a scenario in which  $W_s/\eta = 100$ ,  $\sigma^2 = 1/200$  and where  $W_1 = 1/100$  is the same power for all the interferers. Simulation results are superimposed in both figures onto the results obtained from (8).

In Fig. 3 some numerical results for the one dominant interferer case are plotted for  $q = 1/2$ . The outage probability in (12) is represented as a function of SINR for different values of INR. Also, the approximated values from expression (14) are superimposed in the same figure for the medium and high INR values (i.e. 5 and 15 dB), whereas the interference-free expression in (3) is represented as an approximation for the low INR regime (i.e. -5 dB). On the one hand, this figure shows that (14) fits well with the exact expression (12) in the medium-high INR regime. Besides, it is observed that the approximation is tighter as the INR increases. On the other hand, it is shown that the interference-free approximation is reasonably tight for low INR values. Finally, Fig. 3 shows that noise has a slightly greater impact on outage probability than interference for the low SINR regime.

## VI. CONCLUSIONS

An incomplete generalized MGF of the Hoyt distribution has been studied in this work. This statistical function yields closed-form analytical and easily computable results which are applicable in many realistic cases for Hoyt fading channels. In particular, this mathematical tool is applied to analyze the outage probability for Hoyt fading channels under the presence of background noise and independent Rayleigh interferers with arbitrary powers. Exact closed-form expressions for the outage probability in this scenario have been obtained in the form of finite combinations of Marcum Q, Bessel and elementary functions.

### APPENDIX A PROOF OF LEMMA 1

Since the integrands involving  $\tilde{\mathcal{G}}_X(n, s; \zeta)$  and  $\tilde{\mathcal{G}}_{\langle X \rangle}(n, s; \zeta)$  are bounded and continuous for  $n \geq 0$ , both exist under this

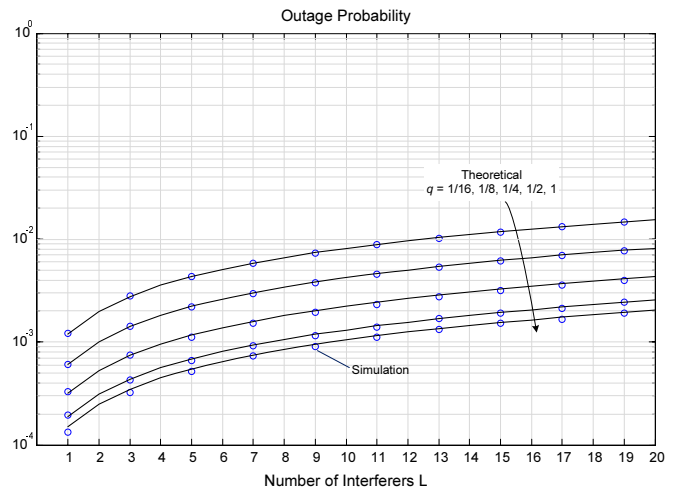


Fig. 2. Outage probability versus number of interferers for an scenario with  $W_s/\eta = 100$ ,  $\sigma^2 = 1/200$  and same power  $W_1 = 1/100$  for all the interferers.

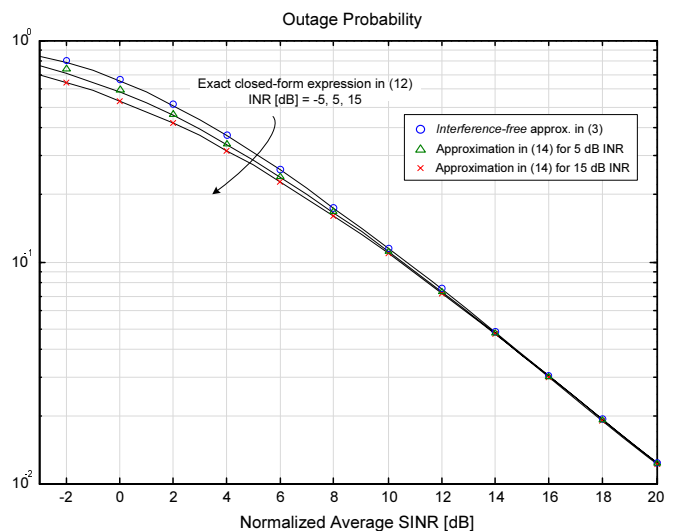


Fig. 3. Outage probability versus normalized average SINR for the one dominant interferer case with  $q = 1/2$ . This plot shows that (14) is a good approximation to (12) in the medium-high INR regime, whereas (3) can be used as an approximation for low INR values.

assumption. Also,  $\mathcal{G}_X(n, s; \zeta)$  exists if  $\mathcal{G}_{\langle X \rangle}(n, s; \zeta)$  does and, considering [7, eq. 6.624-5], this occurs when  $n \geq 0$  and  $\Re\{s\} < \frac{2q^2}{1-q^2}$ . The first equality in (4) is obtained by the change of variable  $\langle X \rangle = \frac{1-q^4}{4q^2\Omega_X} X$  and the second equality is straightforward by [7, eq. 6.624-5] after some simple algebraic manipulations.

### APPENDIX B PROOF OF PROPOSITION 1

The result in this proposition can be obtained by simplifying the general equations derived in [8]. Alternatively, a direct proof is carried out by induction over  $n$  whose key steps are as follows. For  $n = 1$  the identity [9, eq. 5.7] obtained by Agrest and Maksimov is used. The case  $n = 2$  is proved by considering a straightforward modification of the Sonine identity [10, pp. 132] for  $I_\nu(t)$ , and then particularizing for

$\nu = 0$  and  $f(t) = \exp(-\alpha t)$ . Finally, the general case  $n = k \geq 2$  follows from Luke's recursive formulas [11, pp. 120, eq. (5)].

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